CSE 5243 INTRO. TO DATA MINING

Locality Sensitive Hashing

Yu Su, CSE@The Ohio State University

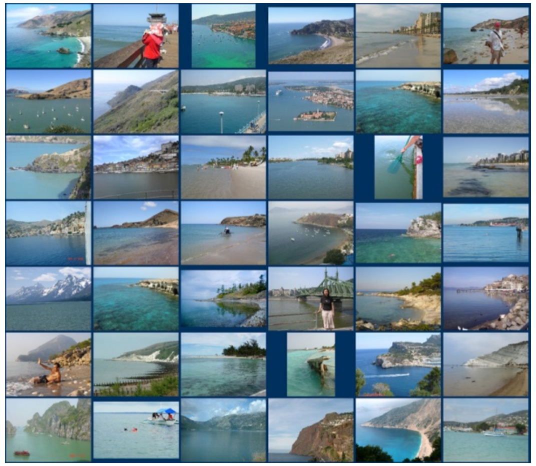
Slides adapted from UIUC CS412 by Prof. Jiawei Han and OSU CSE5243 by Prof. Huan Sun

MMDS Secs. 3.2-3.4.

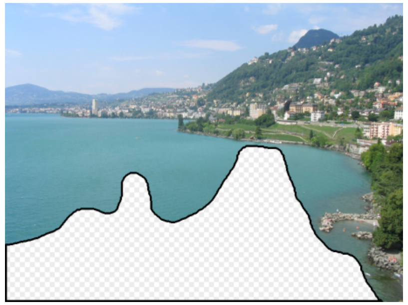
Slides adapted from: J. Leskovec, A. Rajaraman, J. Ullman: Mining of Massive Datasets, http://www.mmds.org

FINDING SIMILAR ITEMS

Scene Completion Problem



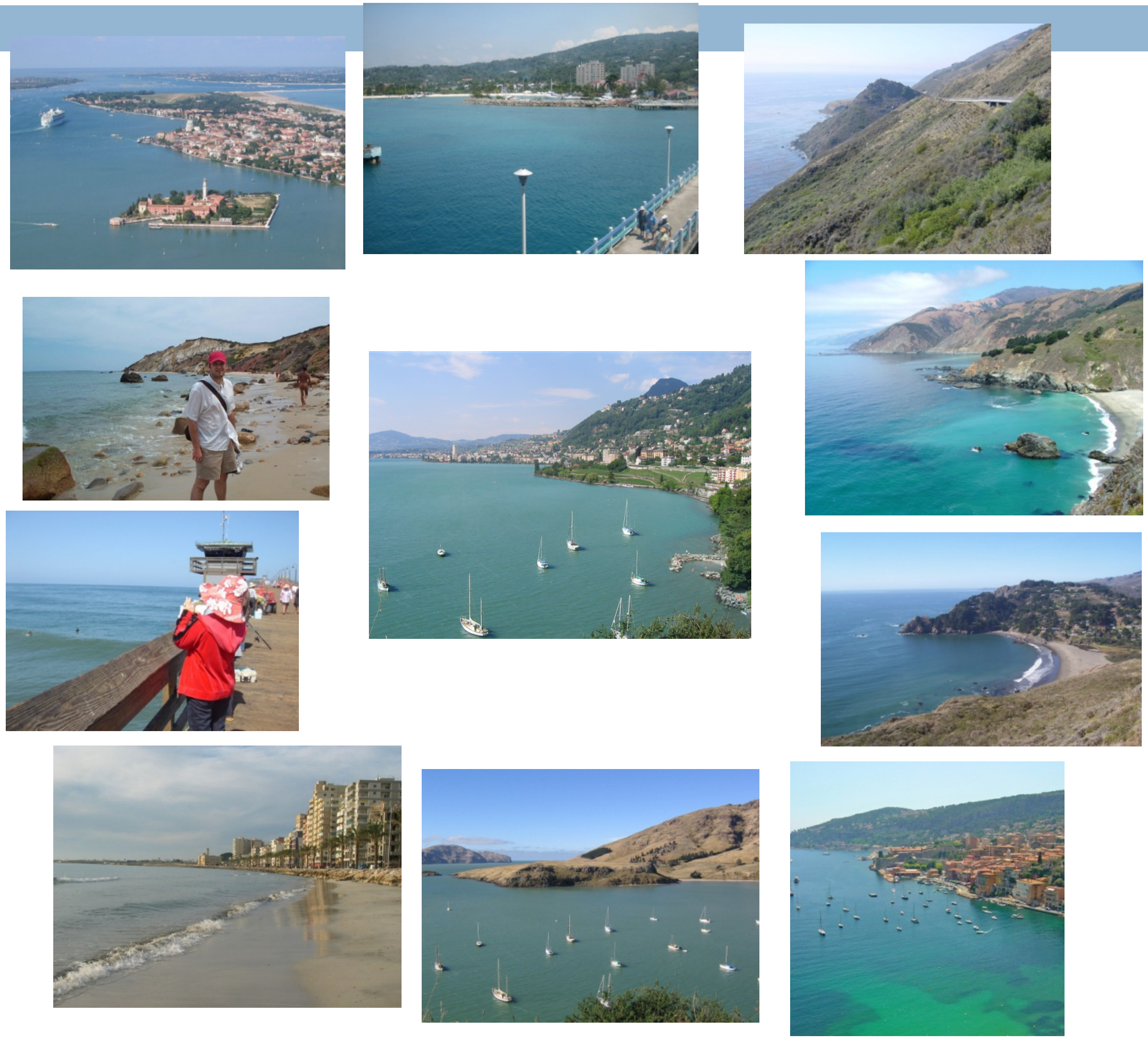
Scene Completion Problem



Scene Completion Problem

10 nearest neighbors from a collection of 20,000 images

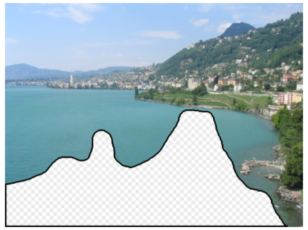
Scene Completion Problem



10 nearest neighbors from a collection of 2 million images

A Common Metaphor

- Many problems can be expressed as finding "similar" sets:
  - **Find near-neighbors in high-dimensional space**
- Examples:
  - **Pages with similar words**
    - For duplicate detection, classification by topic
  - **Customers who purchased similar products**
    - Products with similar customer sets
  - **Images with similar features**
    - Users who visited similar websites



Problem for Today's Lecture

- **Given: High dimensional data points** $x_1, x_2, \dots$
  - **For example:** Image is a long vector of pixel colors

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix} \rightarrow [1\ 2\ 1\ 0\ 2\ 1\ 0\ 1\ 0]$$

- **And some distance function** $\boldsymbol{d(x_1, x_2)}$
  - Which quantifies the "distance" between $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$
- **Goal:** Find **all pairs of data points** (x_i, x_j) that are within some distance threshold $\boldsymbol{d(x_i, x_j) \le s}$
- **Note:** Naïve solution would take $\boldsymbol{O(N^2)}$ ☹

  where $\boldsymbol{N}$ is the number of data points
- **MAGIC: This can be done in** $\boldsymbol{O(N)}$**!! How?**

Task: Finding Similar Documents

- **Goal: Given a large number (N in the millions or billions) of documents, find "near duplicate" pairs**
- **Applications:**
  - Mirror websites, or approximate mirrors → remove duplicates
  - Similar news articles at many news sites → cluster

Task: Finding Similar Documents

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What are the challenges?

Task: Finding Similar Documents

- **Goal: Given a large number (N in the millions or billions) of documents, find "near duplicate" pairs**
- **Applications:**
  - Mirror websites, or approximate mirrors → remove duplicates
  - Similar news articles at many news sites → cluster
- **Problems:**
  - Many small pieces of one document can appear out of order in another
  - Too many documents to compare all pairs
  - Documents are so large or so many that they cannot fit in main memory

Two Essential Steps for Similar Docs

1. ***Shingling:*** Convert documents to sets

2. ***Min-Hashing:*** Convert large sets to short signatures, while preserving similarity

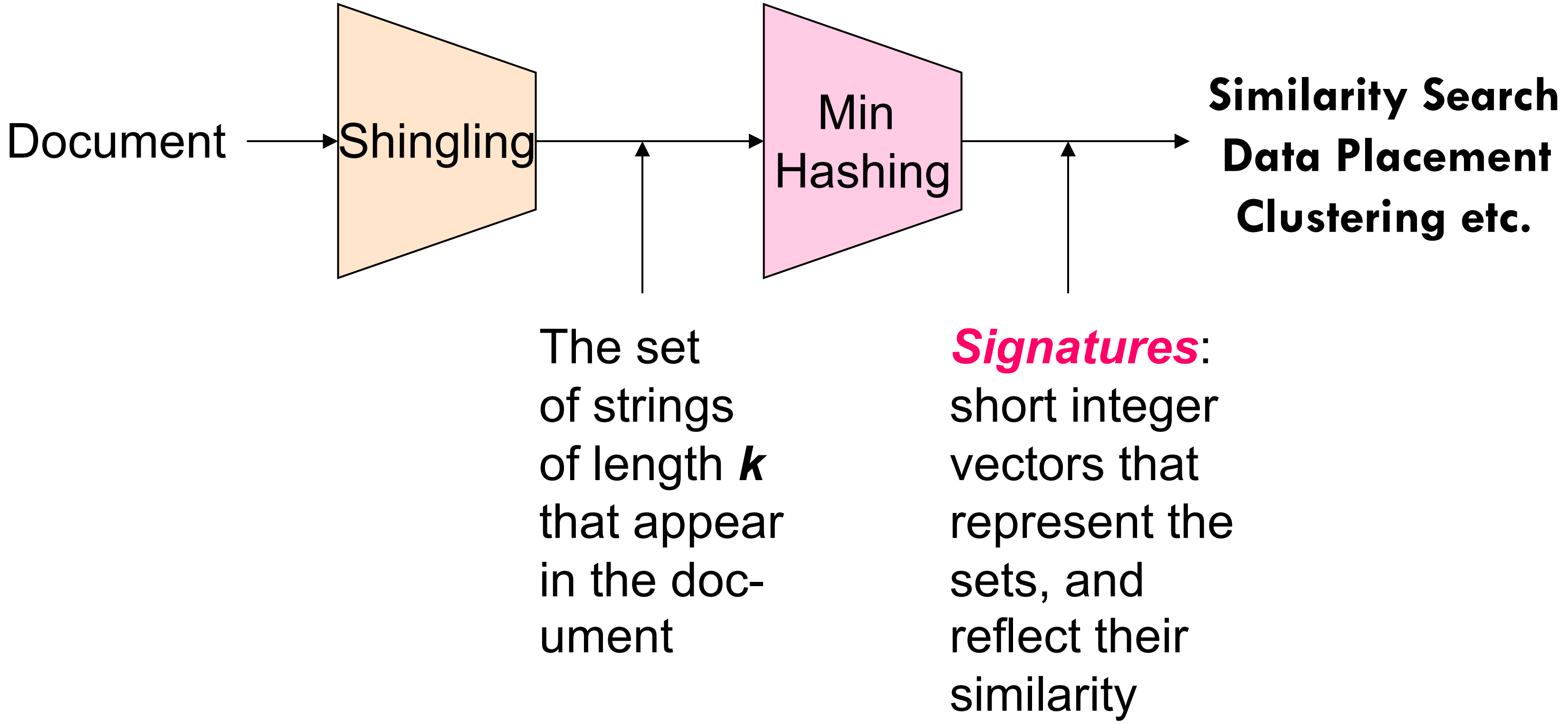
Host of follow up applications

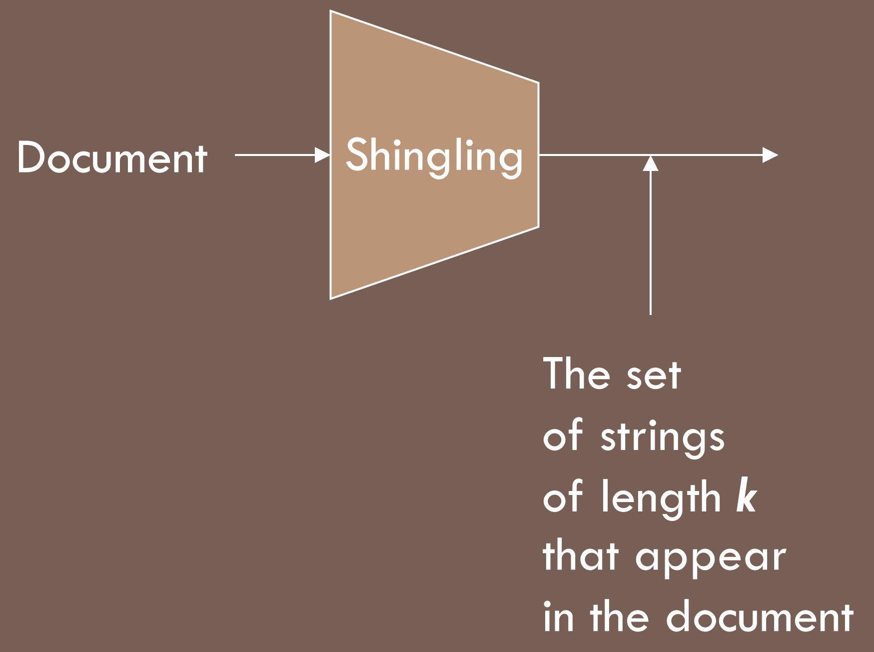
e.g. Similarity Search

Data Placement

Clustering etc.

The Big Picture





SHINGLING

Step 1: ***Shingling:*** Convert documents to sets

Documents as High-Dim Data

- **Step 1: *Shingling:* Convert documents to sets**

- **Simple approaches:**
  - Document = set of words appearing in document
  - Document = set of "important" words
  - Don't work well for this application. Why?

- **Need to account for ordering of words!**
- A different way: **Shingles!**

Define: Shingles

- A *k*-shingle (or *k*-gram) for a document is a sequence of *k* tokens that appears in the doc
  - Tokens can be characters, words or something else, depending on the application
  - Assume tokens = characters for examples

Define: Shingles

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- **Example:** **k=2**; document **D_1** = abcab
  Set of 2-shingles: **$S(D_1)$** = {ab, bc, ca}

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- **Example:** **k=2**; document $\mathbf{D_1}$ = abcab
  Set of 2-shingles: $\mathbf{S(D_1)}$ = {ab, bc, ca}

  - **Another option:** Shingles as a **bag** (multiset), count ab twice: $\mathbf{S'(D_1)}$ = {ab, bc, ca, ab}

Shingles: How to treat white-space chars?

Example 3.4: If we use $k = 9$, but eliminate whitespace altogether, then we would see some lexical similarity in the sentences "`The plane was ready for touch down`". and "`The quarterback scored a touchdown`". However, if we retain the blanks, then the first has shingles `touch dow` and `ouch down`, while the second has `touchdown`. If we eliminated the blanks, then both would have `touchdown`. □

It makes sense to replace any sequence of one or more white-space characters (blank, tab, newline, etc.) by a single blank.

This way distinguishes shingles that cover two or more words from those that do not.

J. Leskovec, A. Rajaraman, J. Ullman: Mining of Massive Datasets, http://www.mmds.org

How to choose K?

- **Documents that have lots of shingles in common have similar text, even if the text appears in different order**

- **Caveat:** You must pick k large enough, or most documents will have most shingles
  - $k = 5$ is OK for short documents
  - $k = 10$ is better for long documents

Compressing Shingles

- To **compress long shingles**, we can **hash** them to (say) 4 bytes
  - Like a Code Book
  - If #shingles manageable → Simple dictionary suffices

e.g., 9-shingle => bucket number [0, 2^32 - 1]

(using 4 bytes instead of 9)

Compressing Shingles

- To **compress long shingles**, we can **hash** them to (say) 4 bytes
  - Like a Code Book
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- **Doc represented by the set of hash/dict. values of its *k*-shingles**
  - **Idea:** Two documents could appear to have shingles in common, when the hash-values were shared

Compressing Shingles

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  - Like a Code Book
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- **Doc represented by the set of hash/dict. values of its *k*-shingles**
- **Example:** **k=2**; document $\mathbf{D_1}$= abcab
  Set of 2-shingles: $\mathbf{S(D_1)}$ = {ab, bc, ca}
  Hash the singles: $\mathbf{h(D_1)}$ = {1, 5, 7}

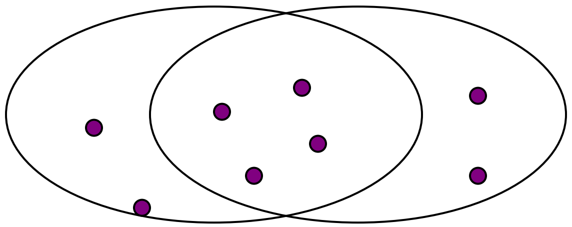
Similarity Metric for Shingles

- **Document D_1 is a set of its k-shingles $C_1=S(D_1)$**

- Equivalently, each document is a 0/1 vector in the space of *k*-shingles
  - Each unique shingle is a dimension
  - Vectors are very sparse

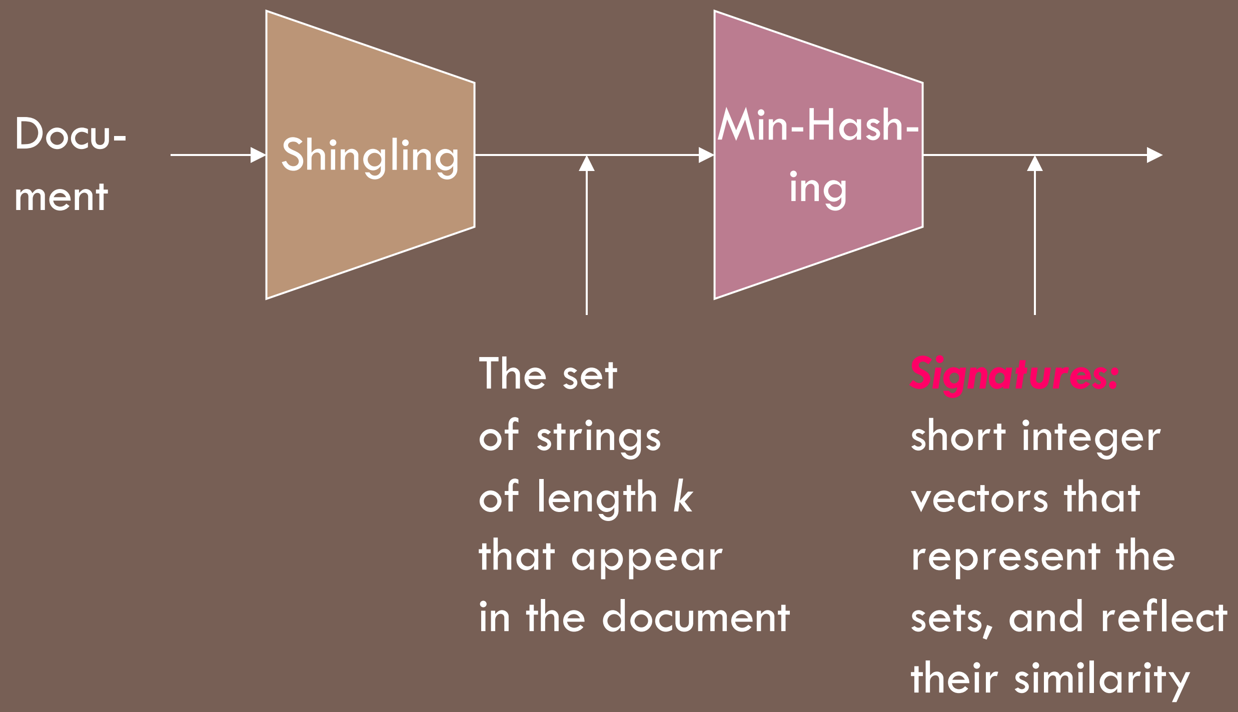
- **A natural similarity measure is the Jaccard similarity:**

$$sim(D_1, D_2) = |C_1 \cap C_2| / |C_1 \cup C_2|$$



Motivation for Minhash/LSH

- **Suppose we need to find similar documents among $N = 1$ million documents**
- Naïvely, we would have to compute **pairwise Jaccard similarities** for **every pair of docs**
  - $N(N-1)/2 \approx$ **5*10¹¹** comparisons
  - At 10^5 secs/day and 10^6 comparisons/sec, it would take **5 days**
- For $N = 10$ million, it takes more than a year…

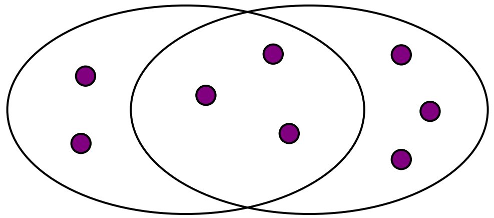


MINHASHING

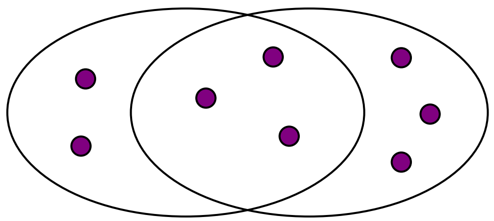
Step 2: ***Minhashing:*** Convert **large variable length sets** to **short <u>fixed-length</u> signatures**, while **<u>preserving similarity</u>**

Encoding Sets as Bit Vectors

- Many similarity problems can be formalized as **finding subsets that have significant intersection**



Encoding Sets as Bit Vectors



- Many similarity problems can be formalized as **finding subsets that have significant intersection**
- **Encode sets using 0/1 (bit, boolean) vectors**
  - One dimension per element in the universal set
- Interpret set intersection as bitwise **AND**, and set union as bitwise **OR**

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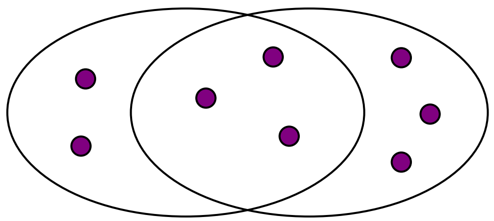

- **Encode sets using 0/1 (bit, boolean) vectors**
  - One dimension per element in the universal set
- Interpret set intersection as bitwise **AND**, and set union as bitwise **OR**
- **Example:** C_1 = 10111; C_2 = 10011
  - Size of intersection = **3**; size of union = **4**,
  - **Jaccard similarity** (not distance) = **3/4**
  - **Distance:** $d(C_1,C_2)$ **= 1 – (Jaccard similarity) = 1/4**

From Sets to Boolean Matrices

- **Rows** = elements (shingles)

Note: Transposed Document Matrix

- **Columns** = sets (documents)
  - 1 in row **e** and column **s** if and only if **e** is a valid shingle of document represented by s
  - Column similarity is the Jaccard similarity of the corresponding sets (rows with value *1*)
  - **Typical matrix is sparse!**

Documents

Shingles

1	1	1	0
1	1	0	1
0	1	0	1
0	0	0	1
1	0	0	1
1	1	1	0
1	0	1	0

Outline: Finding Similar Columns

- **So far:**
  - A documents → a set of shingles
  - Represent a set as a boolean vector in a matrix

$$
\begin{array}{cc}
 & \text{Documents} \\
\text{Shingles} & \left[\begin{array}{cccc}
1 & 1 & 1 & 0 \\
1 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 \\
1 & 1 & 1 & 0 \\
1 & 0 & 1 & 0
\end{array}\right]
\end{array}
$$

Outline: Finding Similar Columns

- **So far:**
  - A documents → a set of shingles
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- **Next goal: Find similar columns while computing small signatures**
  - **Similarity of columns == similarity of signatures**

Documents

Shingles

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0	1	0	1
0	0	0	1
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    - **Essential:** Similarities of signatures and columns are related
  - **3) Optional:** Check that columns with similar signatures are really similar

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    - **Essential:** Similarities of signatures and columns are related
  - **3) Optional:** Check that columns with similar signatures are really similar

- **Warnings:**
  - Comparing all pairs may take too much time: **Job for LSH**
    - These methods can produce false negatives, and even false positives (if the optional check is not made)

Hashing Columns (Signatures) : LSH principle

- **Key idea:** "hash" each column **C** to a small ***signature*** ***h(C)***, such that:
  - (1) ***h(C)*** is small enough that the signature fits in RAM
  - (2) ***sim(C$_1$, C$_2$)*** is the same as the "similarity" of signatures ***h(C$_1$)*** and ***h(C$_2$)***

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- **Goal: Find a hash function $h(\cdot)$ such that:**
  - If $sim(C_1,C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$
  - If $sim(C_1,C_2)$ is low, then with high prob. $h(C_1) \neq h(C_2)$

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- Hash docs into buckets. Expect that "most" pairs of near duplicate docs hash into the same bucket!

Min-Hashing

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- **Clearly, the hash function depends on the similarity metric:**
  - Not all similarity metrics have a suitable hash function

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  - Not all similarity metrics have a suitable hash function

- **There is a suitable hash function for the Jaccard similarity:** It is called **Min-Hashing**

Min-Hashing

- Imagine the rows of the boolean matrix permuted under **random permutation** π

Min-Hashing

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- Define a **"hash" function** $h_\pi(C)$ = the index of the **first** (in the permuted order π) row in which column **C** has value **1**:

$$h_\pi(C) = \min_\pi \pi(C)$$

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Zoo example (shingle size k=1)

Universe $\longrightarrow$ { dog, cat, lion, tiger, mouse}

π_1 $\longrightarrow$ [cat, mouse, lion, dog, tiger]

π_2 $\longrightarrow$ [lion, cat, mouse, dog, tiger]

A = { mouse, lion }

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π_2 $\longrightarrow$ [lion, cat, mouse, dog, tiger]

A = { mouse, lion }

$mh_1(A) = \min(\pi_1\{\text{mouse, lion}\}) = \text{mouse}$

$mh_2(A) = \min(\pi_2\{\text{mouse, lion}\}) = \text{lion}$

Min-Hashing Example

Permutation π

2
3
7
6
1
5
4

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1

Min-Hashing Example

2nd element of the permutation is the first to map to a 1

Permutation π

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3
7
6
1
5
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0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1

Min-Hashing Example

Permutation π

2	4
3	2
7	1
6	3
1	6
5	7
4	5

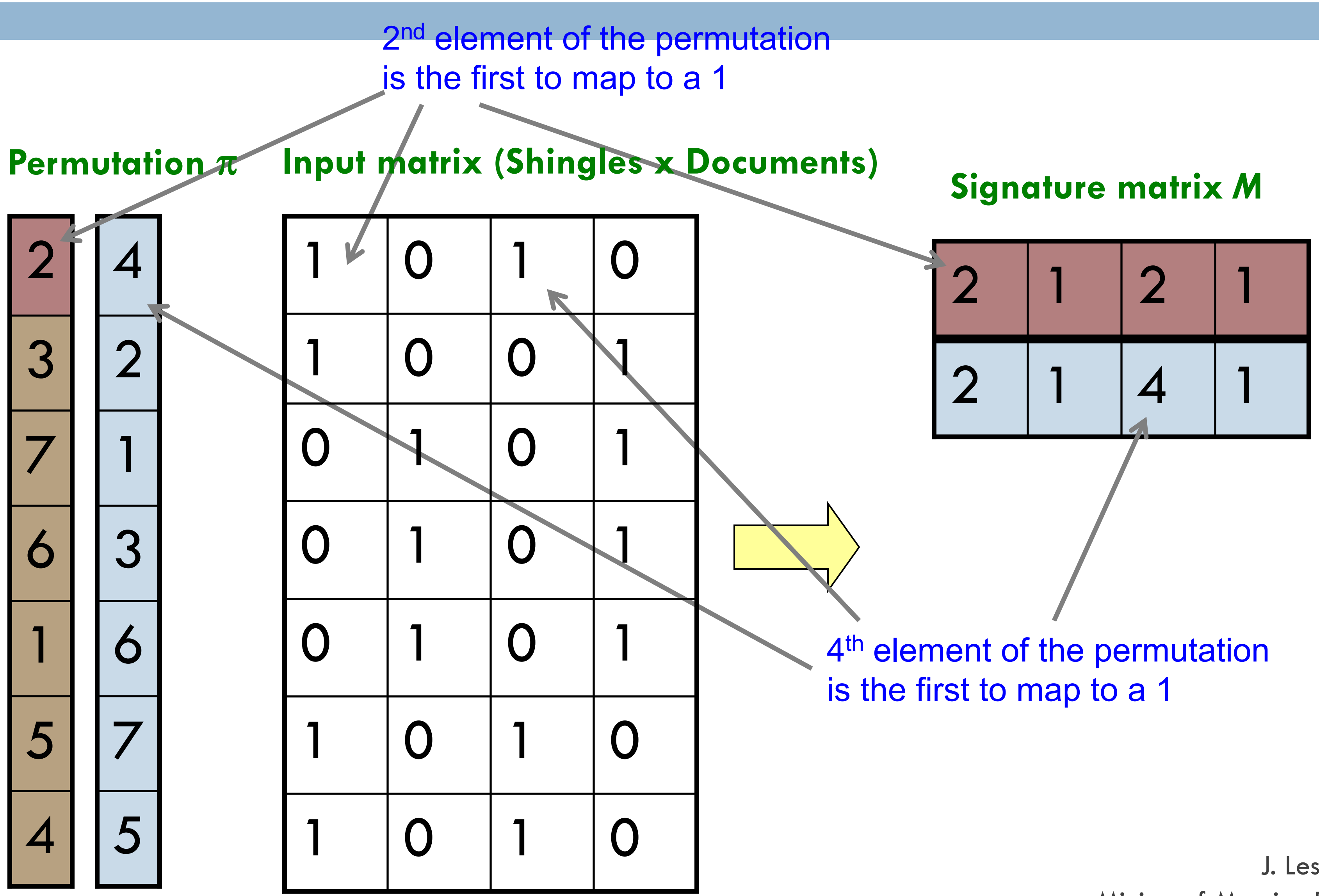
Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1

Min-Hashing Example



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Min-Hashing Example

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

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Min-Hashing Example

Note: Another (equivalent) way is to store row indexes or raw shingles (e.g. mouse, lion):

1	5	1	5
2	3	1	3
6	4	6	4

2nd element of the permutation is the first to map to a 1

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

4th element of the permutation is the first to map to a 1

Min-Hash Signatures

- **Pick K=100 random permutations of the rows**
- Think of ***sig*(C)** as a column vector
- ***sig*(C)[i] =** according to the *i*-th permutation, the index of the first row that has a 1 in column C

$$sig(C)[i] = \min(\pi_i(C))$$

- **Note:** The sketch (signature) of document C is small **~100 bytes!**
- **We achieved our goal! We "compressed" long bit vectors into short signatures**

Key Fact

For two sets A, B, and a min-hash function *$mh_i()$*:

$$Pr[mh_i(A) = mh_i(B)] = Sim(A, B) = \frac{|A \cap B|}{|A \cup B|}$$

Unbiased estimator for *Sim* using ***K*** hashes (notation policy – this is a different K from size of shingle)

$$\hat{Sim}(A, B) = \frac{1}{k} \sum_{i=1:k} I[mh_i(A) = mh_i(B)]$$

Min-Hashing Example

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
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1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

Similarities:

	1-3	2-4	1-2	3-4
Col/Col	0.75	0.75	0	0
Sig/Sig	0.67	1.00	0	0

The Min-Hash Property

0	0
0	0
1	**1**
0	0
0	1
1	0

- **Choose a random permutation π**
- **Claim:** $\Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
- **Why?**

One of the two cols had to have 1 at position ***y***

The Min-Hash Property

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  - Let **X** be a doc (set of shingles), $y \in X$ is a shingle

0	0
0	0
1	**1**
0	0
0	1
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  - **Then:** $\Pr[\pi(y) = \min(\pi(X))] = 1/|X|$
    - It is equally likely that any $y \in X$ is mapped to the ***min*** element

0	0
0	0
1	1
0	0
0	1
1	0

One of the two cols had to have 1 at position ***y***

The Min-Hash Property

0	0
0	0
1	**1**
0	0
0	1
1	0

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  - **Then:** $\Pr[\pi(y) = \min(\pi(X))] = 1/|X|$
    - It is equally likely that any $y \in X$ is mapped to the ***min*** element
  - Let y be s.t. $\pi(y) = \min(\pi(C_1 \cup C_2))$
  - **Then either:** $\pi(y) = \min(\pi(C_1))$ if $y \in C_1$, **or**

    $\pi(y) = \min(\pi(C_2))$ if $y \in C_2$

One of the two cols had to have 1 at position ***y***

The Min-Hash Property

- **Choose a random permutation π**
- **Claim:** $\Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
- **Why?**
  - Let **X** be a doc (set of shingles), $y \in X$ is a shingle
  - **Then:** $\Pr[\pi(y) = \min(\pi(X))] = 1/|X|$
    - It is equally likely that any $y \in X$ is mapped to the *min* element
  - Let y be s.t. $\pi(y) = \min(\pi(C_1 \cup C_2))$
  - **Then either:** $\pi(y) = \min(\pi(C_1))$ if $y \in C_1$, **or**

    $\pi(y) = \min(\pi(C_2))$ if $y \in C_2$
  - So the prob. that **both** are true is the prob. $y \in C_1 \cap C_2$
  - $\Pr[\min(\pi(C_1))=\min(\pi(C_2))]=|C_1 \cap C_2|/|C_1 \cup C_2| = sim(C_1, C_2)$

0	0
0	0
1	1
0	0
0	1
1	0

One of the two cols had to have 1 at position *y*

The Min-Hash Property (Take 2: simpler proof)

- **Choose a random permutation π**
- **<u>Claim:</u> $\Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$**
- **Why?**
  - Given a set X, the probability that any one element is the min-hash under π is $1/|X|$ ← (0)
    - It is equally likely that any $y \in X$ is mapped to the ***min*** element
  - Given a set X, the probability that one of any **k** elements is the min-hash under π is $\mathbf{k}/|X|$ ← (1)
  - For $C_1 \cup C_2$, the probability that any element is the min-hash under π is $1/|C_1 \cup C_2|$ (from 0) ← (2)
  - For any C_1 and C_2, the probability of choosing the same min-hash under π is $|C_1 \cap C_2|/|C_1 \cup C_2|$ ← from (1) and (2)

Similarity for Signatures

- We know: $\Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
- Now generalize to multiple hash functions

- **The *similarity of two signatures* is the fraction of the hash functions in which they agree**

- **Note:** Because of the Min-Hash property, the similarity of columns is the same as the expected similarity of their signatures

Min-Hashing Example

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

Similarities:

	1-3	2-4	1-2	3-4
Col/Col	0.75	0.75	0	0
Sig/Sig	0.67	1.00	0	0

Min-Hash Signatures

- **Pick K=100 random permutations of the rows**
- Think of ***sig*(C)** as a column vector
- ***sig*(C)[i]** = according to the *i*-th permutation, the index of the first row that has a 1 in column C

$$sig(C)[i] = \min(\pi_i(C))$$

- **Note:** The sketch (signature) of document C is small **~100 bytes!**
- **We achieved our goal! We "compressed" long bit vectors into short signatures**

Implementation Trick

- **Permuting rows even once is prohibitive**
- **Approximate Linear Permutation Hashing**
- Pick K independent hash functions (use a, b below)
  - Apply the hash function on ***each column (document)*** for each hash function and get minhash signature

How to pick a random hash function h(x)?

Universal hashing:

$h_{a,b}(x)=((a\cdot x+b) \bmod p) \bmod N$
where:
a,b … random integers
p … prime number (p > N)

Summary: 2 Steps

- **Shingling:** Convert documents to sets
  - We used hashing to assign each shingle an ID

- **Min-Hashing:** Convert large sets to short signatures, while preserving similarity
  - We used **similarity preserving hashing** to generate signatures with property $\mathbf{Pr}[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$

  - We used hashing to get around generating random permutations